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MAXIMUM BENDING MOMENTS FOR MOVING LOADS IN A DRAW BEAM.

BY E. C. MURPHY.

This paper is part of an investigation which the writer has been making of "Maximum bending moments in structures carrying moving loads." Other papers by the writer on this general subject may be found in vol. 6, *Mathematical Messenger*, and vol. 1, *Kansas University Quarterly*.

The bending moment at any assumed point or section of a structure varies as the load passes over the structure. For some position of the load the moment at this point is greater than that for any other portion of the load, but the bending moment at some other point of the structure may be greater than at the assumed point; hence, to find the greatest bending moment which can ever be produced in a given structure by a given load, we must find the position of the load when it produces this maximum moment,* and the point or section of the structure where the maximum moment occurs.

Two kinds of moving loads are considered — a concentrated load and a uniformly distributed load.

CASE I—A CONCENTRATED MOVING LOAD.

Fig. 1 shows a draw beam sustaining its own weight and a concentrated moving load, P; the end spans are of equal length.

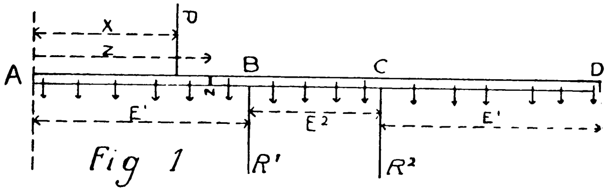

Fig 1

Let $P =$ the concentrated load.

$l = 2l_1 + l_2$.

$q =$ the weight of the beam per linear foot.

$w = ql =$ the weight of the beam.

R_1 and $R_2 =$ the abutment reactions.

$x =$ the distance from left end of beam to load, P.

$z =$ the distance from left end of beam to section considered.

$M_1', M_2', M_3' =$ the moment in the 1st, 2d and 3d spans respectively, when P is in the 1st span.

$M_1'', M_2'', M_3'' =$ the moment in the 1st, 2d and 3d spans respectively, when P is in the 2d span.

$M_1''', M_2''', M_3''' =$ the moment in the 1st, 2d and 3d spans respectively, when P is in the 3d span.

$(M)\text{max.} =$ the maximum moment.

Considering the whole beam as a free body, and taking moments about A, we have—

$$R_1 l_1 + R_2(l_2 + l_1) - Px - \frac{wl}{2} = o. \qquad (1)$$

From Σ (vertical forces) $= o$ we have—

$$R_1 + R_2 - w - P = o \qquad (2)$$

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* Hereafter in this paper the term moment is used in place of bending moment.

From (1) and (2) we find—

$$R_1 = \frac{w}{2} + P\frac{(l_1 + l_2 - x)}{l_2} \text{ and } R_2 = \frac{w}{2} - P\frac{(l_1 - x)}{l_2}.$$

Making a section at any distance z from A, z being less than l and greater than x, and considering the part on the left as a free body, we have—

$$M'_1 = q\frac{z^2}{2} + P(z - x) \tag{3}$$

Differentiating M'_1 with respect to z, and putting the first derivative equal to zero to find the position of the maximum moment, we have—

$$\frac{dM'_1}{dz} = qz + P = o \tag{4}$$

Solving (4) we have $z = -\frac{P}{q}$.

Since z is measured from the left end of the beam, negative values of it do not satisfy the conditions of this problem, and this value must be rejected. From (3) we see that, since the terms containing z are all positive, and the term containing x is negative, M'_1 will be greatest when x is least and z has its greatest value, which are, for this first span, $x = o$ and $z = l_1$. Substituting these values in (3), we have—

$$(M'_1)\text{max.} = q\frac{l_1^2}{2} + Pl_1 \tag{5}$$

For any section of the beam between B and C, *i.e.*, $z > l_1$ and $< l_1 + l_2$, we have—

$$M'_2 = q\frac{z^2}{2} + P(z - x) - R_1(z - l_1) \tag{6}$$

$$\frac{dM'_2}{dz} = qz + P - R_1 = qz + P - q\frac{l}{2} - P\frac{(l_1 + l_2 - x)}{l_2} = o \tag{7}$$

From (7) we find $z = \frac{l}{2} + P\frac{(l_1 - x)}{l_2}$ (8)

Examining (6) and the value of R_1, we find that R_1 is least, and consequently M'_2 is greatest, when x is least, or zero. Substituting this value of x and the corresponding value of z found from (8) we have—

$$(M'_2)\text{max.} = q\frac{l}{4}\left(\frac{l}{2} - l_2\right) + \frac{Pl_1}{2} \tag{9}$$

$$M'_3 = q\frac{z^2}{2} + P(z - x) - R_1(z - l_1) - R_2(z - l_1 - l_2)$$

$$= q\frac{z^2}{2} + Pz - (w + P)(z - l_1) + \frac{wl_2}{2} - Pl_1 \tag{10}$$

Equation (10) being independent of x, the moment in the 3d span is not influenced by the load, P, in the 1st span.

Considering the part on the right of a section in the 3d span as a free body, we have—

$M'_3 = q\left(\frac{l - z}{2}\right)^2$, which is a maximum for $l - z = l_1$; hence,

$$(M'_3)\text{max.} = q\frac{l_1^2}{2} \tag{11}$$

When x has any value greater than l and less than $l_1 + l_2$, that is, when P is in the 2d span, we have—

$$M''_1 = q\frac{z^2}{2} \tag{12}$$

M''_1 is a maximum for $z = l_1$; hence, $(M''_1)\text{max.} = q\frac{l_1^2}{2}$ (13)

$$\mathrm{M}_2'' = q\frac{z^2}{2} + \mathrm{P}(z-x) - \mathrm{R}_1(z-l_1)$$

$$= q\frac{z^2}{2} + \mathrm{P}(z-x) - \Big[\frac{w}{2} + \mathrm{P}\frac{(l_1+l_2-x)}{l_2}\Big](z-l_1) \ldots\ldots \tag{14}$$

Equation (14) has a positive and a negative term containing x; the latter being greater than the former, M_2'' will be greatest when x is least, that is, has the value l_1.

$$\frac{d\mathrm{M}_2''}{dz} = qz + \mathrm{P} - \mathrm{R}_1 = q(z - \frac{l}{2}) - \mathrm{P}\frac{(l_1-x)}{l_2} = o \ldots\ldots \tag{15}$$

Substituting for x its value l_1 in (15), we have $z = \frac{l}{2}$, and substituting these values of x and z in (14), we have—

$$(\mathrm{M}_2'')\mathrm{max.} = q\frac{ll_1}{2} - q\frac{l^2}{8} \ldots\ldots \tag{16}$$

$\mathrm{M}_3'' = q\frac{(l-z)^2}{2}$, which is a maximum for $l - z = l_1$.

$$\text{Hence, } (\mathrm{M}_3'')\mathrm{max.} = q\frac{l_1^2}{2} \ldots\ldots \tag{17}$$

From the symmetry of the beam and loading about the center of the middle span it is evident that—

$$(\mathrm{M}_3''')\mathrm{max.} = (\mathrm{M}_1')\mathrm{max.},\ (\mathrm{M}_1''')\mathrm{max.} = (\mathrm{M}_3')\mathrm{max.},$$
$$\text{and } (\mathrm{M}_2''')\mathrm{max.} = (\mathrm{M}_2')\mathrm{max} \ldots\ldots \tag{18}$$

Comparing the maximum moments in (5), (9), (11), (13), (16), (17), and (18), we see that (M_1')max. is largest; that is, the greatest bending moment which can ever occur in any part of this beam for any position of the moving load, P, is at the supports, and is given by equation (5).

CASE II—A UNIFORMLY DISTRIBUTED MOVING LOAD.

Let the notation be as in case I, except that w = the intensity of the moving load, and x the portion of the beam covered with the load, measured from the left end of the beam.

Taking moments about the left end of the beam, we have—

$$\mathrm{R}_1 l_1 + \mathrm{R}_2(l_1+l_2) - q\frac{l^2}{2} - w\frac{x^2}{2} = o \ldots\ldots \tag{19}$$

From Σ(vertical forces) $= o$ we have—

$$\mathrm{R}_1 + \mathrm{R}_2 - ql - wx = o \ldots\ldots \tag{20}$$

Solving (19) and (20), we have—

$$\left.\begin{array}{l} \mathrm{R}_2 = q\dfrac{l^2 + wx^2}{2l_2} - \dfrac{l_1}{l_2}(ql + wx) \\ \mathrm{R}_1 = -q\dfrac{l^2 + wx^2}{2l_2} + (1 + \dfrac{l_1}{l_2})(ql + wx) \end{array}\right\} \ldots\ldots \tag{21}$$

Proceeding as in case I, we have for the moment in the 1st span—

$$\mathrm{M}_1' = q\frac{z^2}{2} + wx(z - \tfrac{1}{2}x).$$

$$\frac{d\mathrm{M}_1'}{dx} = wx - wx = o,\ x = z.$$

That is, the moment in the 1st span is a maximum when the loading extends up to the section considered.

$$\frac{d\mathrm{M}_1'}{dz} = qz + wx = (q + w)z \ldots\ldots \tag{22}$$

The increment in (22) increases as z increases, and is greatest when $z = l_1$. Hence,

$$(M'_1)\text{max.} = (q + w)\frac{l_1^2}{2} \ldots\ldots\ldots (23)$$

$$M'_2 = q\frac{z^2}{2} + wx(z - \tfrac{1}{2}x) - R_1(z - l_1) \ldots\ldots\ldots (24)$$

$$\frac{dM'_2}{dx} = wz - (1 + \frac{l_1}{l_2})w(z - l_1) - 2x\left[\frac{w}{2} - \frac{w(z - l_1)}{2l_2}\right] \ldots\ldots\ldots (25)$$

Examining (25), we see that the terms not containing x and the coefficient of x are positive; hence, the increment of the moment decreases as x increases, and is greatest when x is least, or zero.

$$\frac{dM'_2}{dz} = qz + wx - (1 + \frac{l_1}{l_2})(ql + wx) + \frac{ql_2 + wx^2}{2l_2} \ldots\ldots\ldots (26)$$

Substituting for x its value, o, in (26), we have $z = \frac{l}{2}$, and these values of x and z in (24) give—

$$(M'_2)\text{max.} = q\frac{ll_1}{4} \ldots\ldots\ldots (27)$$

$M'_3 = q\frac{(l - z)^2}{2}$, which is a maximum for $l - z = l_1$.

$$\text{Hence, } (M'_3)\text{max.} = q\frac{l_1^2}{2} \ldots\ldots\ldots (28)$$

For the load covering the 1st span and part of the 2d, we have—

$$M''_1 = (q + w)\frac{z^2}{2}, \text{ and } (M''_1)\text{max.} = (q + w)\frac{l_1^2}{2} \ldots\ldots\ldots (29)$$

$$M''_2 = q\frac{z^2}{2} + w(z - \tfrac{1}{2}x) - R_1(z - l_1) \ldots\ldots\ldots (30)$$

Equation (30) has the same form as (24), but x in (30) may vary from l_1 to $l_1 + l_2$, while x in (24) can only vary from o to l_1.

Examining (25), which shows how the increment of M'_2 varies with respect to x, we see that, if $l_1 + z^1$ be substituted for z, and $l_1 + x^1$ for x, it reduces to the form $\frac{dM''_2}{dx^1} = -wx^1(1 - \frac{z^1}{l_2})$. This increment being negative, the moment decreases as x increases, and will be greatest when $x = l_1$.

Substituting this value of x in (26) and solving for z, we have—

$z = \frac{l}{2} + \frac{wl_1^2}{2ql_2}$. Hence,

$$(M''_2)\text{max.} = \frac{q}{8}\left(l + \frac{wl_1^2}{ql_2}\right)^2 + \frac{w}{2}\left(l - l_1 + \frac{wl_1^2}{ql_2}\right) - \left(\frac{l_2}{2} + \frac{wl_1^2}{2ql_2}\right)$$
$$\left[(1 + \frac{l_1}{l_2})(ql + wl) - \frac{ql^2 + wl_1^2}{2l_2}\right] \ldots\ldots\ldots (31)$$

$$M''_3 = q\frac{(l - z)^2}{2}, \text{ and } (M''_3)\text{max.} = q\frac{l_1^2}{2} \ldots\ldots\ldots (32)$$

When the loading covers two spans and part of the 3d—

$$M'''_1 = (q + w)\frac{z^2}{2}, \text{ and } (M'''_1)\text{max.} = (q + w)\frac{l_1^2}{2} \ldots\ldots\ldots (33)$$

$$M'''_2 = (q + w)\frac{z^2}{2} - R_1(z - l_1)$$
$$= (q + w)\frac{z^2}{2} - (z - l_1)\left[(1 + \frac{l_1}{l_2})(ql + wx) - \frac{ql^2 + wx^2}{2l_2}\right] \ldots\ldots\ldots (34)$$

$$\frac{dM_2'''}{dx} = (z - l_1)\left[\frac{w(l_2 + l_1) - wx}{l_2}\right] \quad \ldots\ldots (35)$$

From (35) we see that the increment of M_2''' is negative, as x varies from $l_1 + l_2$ to l; hence, M_2''' is greatest for $x = l_1 + l_2$.

$$\frac{dM_2'''}{dz} = (q + w)z - R_1 = o.$$

$$z = \left(1 + \frac{l_1}{l_2}\right)\left[ql + w(l_1 + l_2)\right] - \frac{ql_2 + w(l_1 + l_2)^2}{2l_2} \quad \ldots\ldots (36)$$

The value of z in (36), and the corresponding value of x substituted in (34), will give the value of (M_2''')max.

$$M_3''' = (q + w)\left(\frac{l - z}{2}\right)^2 \text{ and } (M_3''')\text{max.} = (q + w)\frac{l_1^2}{2} \quad \ldots\ldots (37)$$

THE EXTRACTIVE YIELD OF VARIOUS IMPORTANT VEGETABLE MEDICINAL SUBSTANCES.

BY PROF. L. E. SAYRE, UNIVERSITY OF KANSAS.

The amount of extractive yielded by a fibrous drug, while it does not afford a sure test of its value, furnishes a data by which the working pharmacist and physician may judge of the quality of the material in hand. It also gives a means by which such adulterations as insoluble woody fiber may be detected. There have not been to my knowledge any reliable tables made out of this kind, and at the suggestion of a few medical colleagues, I have prepared the following table, which is the result of my own examination of the standard drugs of the market.

Table showing the percentage of extractive, dried at 100° C., contained in prominent medicinal vegetable substances of the market.

(The solvents employed being those of the United States Pharmacopœia.)

	Per Cent.		*Per Cent.*
Aconite	14.15	Hydrangia	9.06
Apocynum cannabinum	14.40	Ipecac root	12.45
Belladonna leaves	22.70	Convalluria	33.50
Burdock root	24.00	Mezereum	11.80
Buchu leaves	23.30	Nux vomica	13.70
Belladonna root	9.65	Podophyllum	13.80
Colchicum root	19.45	Poke root	17.12
Cannabis indica	13.00	Pleurisy root	25.20
Coca leaves	24.70	Stramonium seed	14.10
Conium	28.60	Veratrum	18.00
Digitalis leaves	34.10	Viburnum opulus	15.20
Gelseminum root	11.60	Yerba santa	31.95
Hydrastis	23.20	Juborandi	23.58
Hyoscyamus leaves	19.50		